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Electromagnetic transition in waveguide with application to lasers

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Abstract. The electromagnetic transition of two-level atomic systems in a waveguide is calculated. Compared with the result in free space, the spontaneous emission rate decrease because the phase space is smaller, and meanwhile, some resonance appears in some cases. Moreover, the influence of non-uniform electromagnetic field in a waveguide on absorption and stimulated emission is considered. Applying the results to lasers, a method to enhance the laser power is proposed.

PACS. 42.50.Ct Quantum description of interaction of light and matter; related experiments – 42.55.Ah General laser theory

1 Introduction

The behavior of atoms in confined space had been studied for many years since the possibility of modifying spontaneous emission rates was first mentioned by Purcell [1]. Many studies had been done in theoretical and experimental aspects [2–4]. The transition rate of atoms may increase [1] or decrease [5] in a cave. In this paper, we will discuss spontaneous emission, absorption and stimulated emission of atoms in a waveguide and apply the results to the problem of how to enhance the output power of lasers.

In confined space, the values of momenta are discrete at certain directions and a nonzero lower limit exists due to the boundaries. Compared with free space, the phase space in confined space is smaller. Some processes, *e.g.* atomic decay, will be influenced. In the case of spontaneous emission, the transition rate of excited atoms is determined by two factors: the transition matrix element between two states and the phase space volume of final state. In confined space, the transition matrix element is the same as in free space under the first order approximation. The phase space volume is smaller because some components of photon momenta in final state are constrained to some discrete values. It leads to the decrease of the total spontaneous emission rate of excited atoms. On the other hand, when the frequency of the photon in final state equals to one of the eigenfrequencies of the cave, resonance will occur and the transition rate will remarkably increase. In this paper, we will give a direct calculation about the spontaneous emission rate of excited atoms in a matrix waveguide. Also, the ratio of the spontaneous emission rate in the waveguide to the case in free space is given. The ratio is determined only by the shape of the boundary, but it is independent of the form of the transition matrix. This means that the ratio is not related to special transition processes. Besides, since the electromagnetic field has a defined non-uniform distribution in a waveguide, the rates of absorption and stimulated emission will depend on the position of the atoms. The corresponding transition rate at each position in the waveguide and the mean transition rate are also given.

The power of lasers is related to the spontaneous-emission lifetime of the excited states. The lifetime of atoms can be influenced in confined space. Therefore, we can change the laser power by putting the atoms in a cavity. In the last part of this paper, we will discuss the change of the output power of lasers under the influence of the scale of waveguides.

2 Spontaneous emission in a waveguide

For one-photon decay of an initial state $|a\rangle$ into a final state $|b\rangle$ and a photon with frequency ω_n and polarization σ, the S-matrix element to first order in perturbation theory is (Here we use Heaviside's units and take $c = \hbar = 1$) [6]:

$$S_{ba} = -\mathrm{i}2\pi\delta(E_b + \omega_n - E_a)\frac{1}{\sqrt{L_x L_y L_z}} f_{ba}, \qquad (1)$$

where E_a and E_b are the energies of the two states of the atomic system. f_{ba} is the transition amplitude. Under the dipole approximation, the amplitude takes the form:

$$f_{ba} = -\mathrm{i}e\sqrt{\frac{\omega_n}{2}}\epsilon_n^{\sigma *} \cdot \mathbf{r}_{ba}. \qquad (2)$$

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The vectors ϵ are polarization vectors and $\mathbf{r}_{ba}$ is the matrix element

$$\mathbf{r}_{ba} = \langle b|\mathbf{r}|a\rangle \cdot \tag{3}$$

Consider a two-level system placed in a waveguide with sides L_x and L_y which are comparable with the wavelength of the photon. In the waveguide, the components of the momentum at x and y directions of the photon are constrained to some discrete values:

$$k_x = \frac{n_x\pi}{L_x}, \quad k_y = \frac{n_y\pi}{L_y}, \quad (n_x, n_y = 0, 1, 2, \cdots). \tag{4}$$

Summing over all final photon states, the total spontaneous emission rate is

$$W_{\rm sp} = \sum_{n_x n_y n_z} \frac{1}{L_x L_y L_z} 2\pi\delta(E_b + \omega_n - E_a)|f_{ba}|^2. \tag{5}$$

Since the photon is free at z-direction, the summation of n_z can be replaced by integration:

$$\begin{aligned} W_{\rm sp} &= \sum_{n_x n_y} \frac{1}{L_x L_y} \int \frac{{\rm d}k_z}{2\pi} 2\pi\delta(E_b + \omega_n - E_a)|f_{ba}|^2 \\ &= \sum_{n_x n_y} \frac{1}{L_x L_y} \frac{\omega}{k_0}|f_{ba}|^2 \\ &= \sum_{n_x n_y} \frac{1}{L_x L_y} \frac{e^2\omega^2}{2k_0}|\mathbf{r}_{ba}|^2 \sin^2\theta, \end{aligned} \tag{6}$$

where k_0 is the z-component of wave vector $\mathbf{k}$

$$k_0 = \sqrt{\omega^2 - k_x^2 - k_y^2} \tag{7}$$

and θ is the angle between the dipole moment and wave vector $\mathbf{k}$.

In free space, the total spontaneous emission rate is known as [6]

$$W_{\rm sp}^{\rm free} = \frac{e^2\omega^3}{3\pi}|\mathbf{r}_{ba}|^2. \tag{8}$$

Comparing with equation (6), we obtain the ratio of the rates in confined space and free space:

$$\eta = \frac{W_{\rm sp}}{W_{\rm sp}^{\rm free}} = \sum_{n_x n_y} \frac{1}{L_x L_y} \frac{3\pi}{2\omega k_0} \sin^2\theta. \tag{9}$$

Because the space in a waveguide is not homogeneous, a new factor $\sin^2\theta$ is appeared. This result shows that the spontaneous emission rate is different if the polarization of atoms in the waveguide is different. However, atoms usually are non-polarized, so a mean ratio is needed.

Use (α, β) to describe the direction of $\mathbf{r}_{ba}$ in the waveguide. Since

$$\begin{aligned} \cos\theta &= \frac{\mathbf{k}\cdot\mathbf{e}_r}{|\mathbf{k}|} \\ &= \frac{k_x \sin\alpha\cos\beta + k_y \sin\alpha\sin\beta + k_z\cos\alpha}{|\mathbf{k}|}, \end{aligned} \tag{10}$$

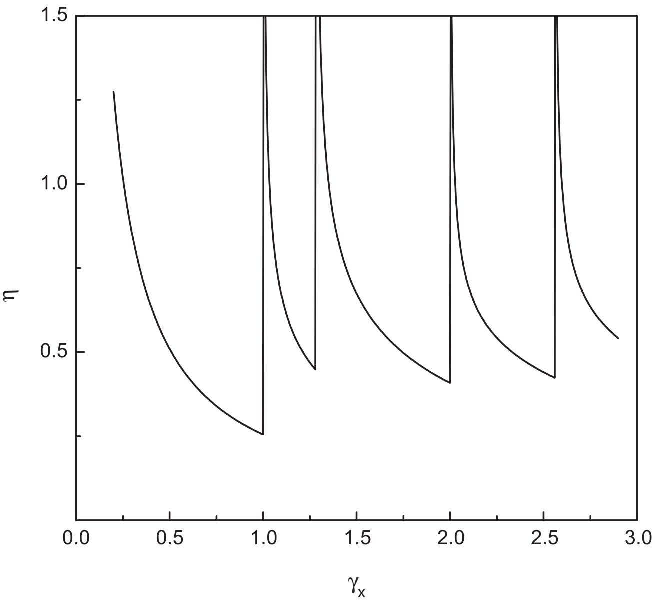

Fig. 1. In a waveguide, the decay rate is usually depressed since the phase space is smaller than in free space, but remarkably increases when resonance occurs ($\gamma_y = 1.6$).

the mean ratio can be obtained as

$$\begin{aligned} \bar{\eta} &= \frac{1}{4\pi} \int \eta \, {\rm d}\Omega \\ &= \sum_{n_x n_y} \frac{1}{L_x L_y} \frac{\pi}{\omega k_0}\cdot \end{aligned} \tag{11}$$

Obviously, this result is only related with three length scales: the wavelength of the photon λ, the widths of the waveguide L_x and L_y. Introducing two parameters:

$$\gamma_x = \frac{L_x}{\lambda/2}, \qquad \gamma_y = \frac{L_y}{\lambda/2}, \tag{12}$$

we can get the following expression:

$$\bar{\eta} = \sum_{n_x n_y} \frac{1}{\pi} \frac{1}{\sqrt{\gamma_x^2\gamma_y^2 - \gamma_y^2 n_x^2 - \gamma_x^2 n_y^2}} \cdot \tag{13}$$

This result shows that the spontaneous emission rate is usually smaller in a waveguide than in free space because the phase space of the final state is smaller and the possible states of photon are fewer. However, the spontaneous emission rate will diverge when γ_x and γ_y take such values that the denominator of (13) is zero. From (7) we can see that this is just the case that the z-component of photon k_z or k_0 vanishes. In other words, when $k_0 = 0$, some resonance occurs and the spontaneous emission rate of excited atoms increases rapidly (see Fig. 1).

Besides, when both γ_x and γ_y are smaller than 1 (see Fig. 2), the frequency of the photon is lower than the lowest permitted frequency in such a waveguide. It shows that the phase space of the final state is compressed to zero so the photon can not be emitted and the transition

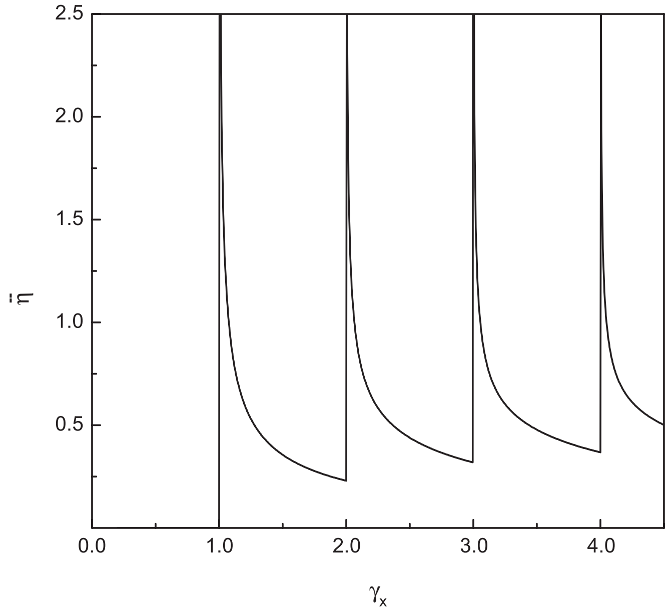

Fig. 2. When $\gamma_x, \gamma_y < 1$, the decay is forbidden (here $\gamma_y = 0.8$).

can not occur. At that time, an excited atom will never return to its ground state, *i.e.*, the transition is forbidden.

3 Absorption and stimulated emission in a waveguide

The rates of absorption and stimulated emission in confined space are different from those in free space because the mode structures of electromagnetic fields are different in the two cases. The processes of absorption and stimulated emission are transitions of atomic systems in external electromagnetic fields. The electromagnetic field in a waveguide has a defined distribution which is not uniform, so the transition probabilities will depend on the position of the atoms.

In a matrix waveguide, the expressions for the spatial components of the field are

$$\begin{cases} E_x(x,y,z) = E_1 \cos k_x x \sin k_y y \mathrm{e}^{\mathrm{i}k_z z}, \\ E_y(x,y,z) = E_2 \sin k_x x \cos k_y y \mathrm{e}^{\mathrm{i}k_z z}, \\ E_z(x,y,z) = E_3 \sin k_x x \sin k_y y \mathrm{e}^{\mathrm{i}k_z z}, \end{cases} \quad (14)$$

where $k_x, k_y, k_z (= k_0)$ have been given in (4) and (7), and E_1, E_2, E_3 satisfy $k_x E_1 + k_y E_2 - \mathrm{i}k_z E_3 = 0$.

We will first consider the processes of absorption in the waveguide. In this case, a monochromatic electromagnetic wave at frequency ω is made to interact with an atom which is in state $|b\rangle$. Since the magnitudes of the atomic electron radii are much smaller than the wavelength of the electromagnetic wave, the dipole approximation is extremely good. On this assumption, we regard $\mathbf{E}$ as an uniform field in a small region and use its value at $z = 0$. The corresponding electric field is a sinusoidal function of time $\mathbf{E}(x,y,t) = \mathbf{E}_0(x,y)\sin\omega t$, where $\mathbf{E}_0(x,y) = E_x(x,y,0)\mathbf{i} + E_y(x,y,0)\mathbf{j} + E_z(x,y,0)\mathbf{k}$. Similar to the case of absorption occurring in free space [7], the rate of absorption can be obtained directly:

$$W_{ab}(x,y) = \frac{\pi}{3n^2}|\mathbf{r}_{ab}|^2 \rho(x,y)\delta(\Delta\omega). \quad (15)$$

Here n is the refractive index of the atomic system; $\Delta\omega$ is the magnitude of the spread in ω; $\mathbf{r}_{ab}$ is the matrix element corresponding to the transition from the initial state $|b\rangle$ to the final state $|a\rangle$; ρ is the energy density of the incident electromagnetic wave in the waveguide:

$$\rho(x,y) = \frac{1}{2}n^2 E_0(x,y)^2 = \frac{1}{2}n^2(E_1^2 \cos^2 k_x x \sin^2 k_y y + E_2^2 \sin^2 k_x x \cos^2 k_y y + E_3^2 \sin^2 k_x x \sin^2 k_y y). \quad (16)$$

Obviously, the transition probability depends on the position of the atom, since the electromagnetic field in waveguide has a non-uniform distribution. The mean rate of absorption in the waveguide is

$$\overline{W}_{ba} = \frac{\pi}{3n^2}|\mathbf{r}_{ba}|^2 \bar{\rho}\delta(\Delta\omega), \quad (17)$$

with the mean energy density

$$\bar{\rho} = \frac{1}{V}\int_V \rho(x,y)\mathrm{d}V = \frac{n^2}{8}(E_1^2 + E_2^2 + E_3^2). \quad (18)$$

The rate of stimulated emission W_{ba} is equal to the rate of absorption W_{ab}, since the transition matrix element $|\mathbf{r}_{ba}| = |\mathbf{r}_{ab}|$ and the photons of stimulated emission must always match the mode of the stimulating photons.

4 Application to lasers

One of the central problems in the area of laser is how to enhance the laser power. There are many factors which can influence the laser power. One of them is the spontaneous-emitted lifetime of excited states. According to the results above, the spontaneous emission rate of excited atoms in confined space, *e.g.* waveguide, is different to the case of free space. This means that we can change the lifetime of excited atoms by adjusting the shape and scale of the cavity (in this paper, the cavity is a waveguide). One factor which directly influences the laser power is the lifetime of the upper laser level. If we put the laser medium in a waveguide, we can enhance or reduce the laser power by changing the lifetime of the upper laser level.

Take a three-level laser proceeds as an example, the output power of continuous wave laser under the rate-equation approximation is [7]

$$P = A\frac{\chi - 1}{\tau} \cdot \quad (19)$$

Here A is a parameter determined by the character of the specific laser medium and laser facility. The quantity τ is the lifetime of the upper laser level, and it is, in general, given by

$$\frac{1}{\tau} = \frac{1}{\tau_{\rm sp}} + \frac{1}{\tau_{\rm nr}}, \tag{20}$$

where $\tau_{\rm sp}$ is the spontaneous-emission lifetime and $\tau_{\rm nr}$ the nonradiative lifetime of upper laser level. Let $W_{\rm p}$ be the pumping rate and $W_{\rm cp}$ the critical pumping rate. The χ is the amount by which threshold is exceeded:

$$\chi = \frac{W_{\rm p}}{W_{\rm cp}} \cdot \tag{21}$$

In practice, the following approximation can be taken:

$$W_{\rm p} \simeq \frac{1}{\tau} \cdot \tag{22}$$

Then the output power is given as

$$P = A\left(W_{\rm p} - \frac{1}{\tau_{\rm nr}} - \frac{1}{\tau_{\rm sp}}\right) \cdot \tag{23}$$

If the process occurs in a waveguide, $\tau_{\rm sp}$ will be replaced by $\tau_{\rm sp}^{\rm W}$. The $\tau_{\rm sp}^{\rm W}$ is the spontaneous-emission lifetime in the waveguide. From equation (9), we can obtain

$$\eta = \frac{W_{\rm sp}}{W_{\rm sp}^{\rm free}} = \frac{1/\tau_{\rm sp}^{\rm W}}{1/\tau_{\rm sp}} = \frac{\tau_{\rm sp}}{\tau_{\rm sp}^{\rm W}} \cdot \tag{24}$$

Substitute this relation into equation (23), the laser power in a waveguide $P^{\rm W}$ is therefore

$$P^{\rm W} = A\left(W_{\rm p} - \frac{1}{\tau_{\rm nr}} - \frac{\eta}{\tau_{\rm sp}}\right) \cdot \tag{25}$$

When $\eta < 1$, the output power enhances, *i.e.* $P^{\rm W} > P$. On the contrary, when $\eta > 1$, the output power reduces, *i.e.* $P^{\rm W} < P$. The analysis carried out here implies that the output power can be adjusted by changing the scale of the waveguide.

Besides, in waveguides, by adjusting the spontaneous-emission lifetime of the upper laser level $\tau_{\rm sp}^{\rm W}$, we can obtain a high power laser pulse. If $\tau_{\rm sp}^{\rm W}$ is long enough, after sufficient pumping, a large population inversion will be obtained. For instance, the spontaneous emission of the upper laser level can be forbidden to a certain extent by modifying the scale of the waveguide. When the population inversion is large enough, change the waveguide to a proper scale to induce a spontaneous transition of the upper laser level. In this way, a laser pulse is obtained.

In conclusion, we have shown that transition rates of atoms may change in confined space. In some cases, the lifetime can be changed by adjusting the experimental arrangements. Based on this idea, a method to enhance the laser power is proposed.

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